EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Show that $\ln (4 \times 12 \times 36 \times 108 \times \ldots\ldots.$ up to n terms) $= 2n \ln 2 + \frac{n(n-1)}{2} \ln 3$.

2. There are n AM's between 1 & 31 such that 7th mean : $(n-1)^{th}$ mean = 5 : 9, then find the value of n.

3. Prove that the average of the numbers n sin n°, n = 2, 4, 6 ,......., 180 is cot 1°.

4. If S be the sum, P the product & R the sum of the reciprocals of n terms of a GP, find the value of $P^2\left(\frac{R}{S}\right)^n$.

5. In a set of four numbers, the first three are in GP & the last three are in AP, with common difference 6. If the first number is the same as the fourth, find the four numbers.

6. Find the sum of the series, 7+77+777 +... to n terms.

7. Find three numbers a, b, c between 2 & 18 such that ;
(i) their sum is 25
(ii) the numbers 2, a, b are consecutive terms of an AP &
(iii) the numbers b, c, 18 are consecutive terms of a GP.

8. If one AM 'a' & two GM's p & q be inserted between any two given numbers then show that $p^3 + q^3 = 2\,apq$.

9. The 1^{st}, 2^{nd} and 3^{rd} terms of an arithmetic series are a, b and a^2 where 'a' is negative. The 1^{st}, 2^{nd} and 3^{rd} terms of a geometric series are a, a^2 and b find the
(i) value of a and b

(ii) sum of infinite geometric series if it exists. If no then find the sum to n terms of the GP.

(iii) sum of the 40 term of the arithmetic series.

10. If the 10th term of an HP is 21 & 21^{st} term of the same HP is 10, then find the 210^{th} term.

11. Find the sum of the n terms and to infinity of the sequence $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots\ldots..$

12. An AP & an HP have the same first term, the same last term & the same number of terms; prove that the product of the r^{th} term from the beginning in one series & the r^{th} term from the end in the other is independent of r.

13. Find the sum of the first n terms of the sequence:
$$1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + 4\left(1+\frac{1}{n}\right)^3 + \ldots\ldots.$$

14. Find the nth term and the sum of n terms of the sequence
(i) 1 + 5 + 13 + 29 + 61 + ...

(ii) 6 + 13 + 22 + 33 + ...

15. The AM of two numbers exceeds their GM by 15& HM by 27. Find the numbers.

16. Sum the following series to n terms and to infinity:

(i) $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \ldots.$

(ii) $\sum_{r=1}^{n} r\,(r+1)\,(r+2)\,(r+3)$

(iii) $\sum_{r=1}^{n} \frac{1}{4r^2-1}$

(iv) $\frac{1}{4} + \frac{1.3}{4.6} + \frac{1.3.5}{4.6.8} + \ldots\ldots\ldots.$.

17. Evaluate the sum $\sum_{n=1}^{\infty} \frac{n^2}{6^n}$.

18. If the sum
$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}} + \ldots. \ldots\ldots + \sqrt{1+\frac{1}{(1999)^2}+\frac{1}{(2000)^2}}$$
equal to n −1/n where n ∈ N. Find n.

19. Show that in any arithmetic progression a_1, a_2, a_3....
$a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots\ldots\ldots + a^2_{2K-1} - a^2_{2K} = [K/(2K - 1)] (a_1^2 - a^2_{2K})$.

20. If the first 3 consecutive terms of a geometrical progression are the roots of the equation
$2x^3 - 19x^2 + 57x - 54 = 0$ find the sum to infinite number of terms of G.P.

21. If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c & all the roots.

22. If a, b, c, d, e be 5 numbers such that a, b, c are in AP; b, c, d, are in GP & c, d, e are in HP then
(i) Prove that a, c, e are in GP.

(ii) Prove that $e = (2b - a)^2/a$.

(iii) If a = 2 & e = 18, find all possible values of b, c, d.

23. If n is a root of the equation
$x^2 (1 - ac) - x (a^2 + c^2) - (1 + ac) = 0$ & if n HM's are inserted between a & c, show that the difference between the first & the last mean is equal to ac (a – c).

24. (i) The value of x + y + z is 15 if a, x, y, z, b are in AP while the value of (1/x) + (1/y) + (1/z) is 5/3 if a, x, y, z, b are in HP. Find a & b.

(ii) The values of xyz is 15/2 or 18/5 according as the series a, x, y, z, b is an AP or HP. Find the values of a & b assuming them to be positive integer.

25. Prove that the sum of the infinite series
$\frac{1\cdot 3}{2} + \frac{3\cdot 5}{2^2} + \frac{5\cdot 7}{2^3} + \frac{7\cdot 9}{2^4} + \ldots\ldots\ldots\ldots \infty = 23$.

26. Find the condition that the roots of the equation $x^3 + px^2 + qx - r = 0$ may be in A.P. and hence solve the equation $x^3 - 12x^2 + 39x - 28 = 0$.

27. If a, b, c be in GP & $\log_c a$, $\log_b c$, $\log_a b$ be in AP, then show that the common difference of the AP must be 3/2.

28. Two distinct, real infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is 1/8. If the second term of both the series can be written in the form $\frac{\sqrt{m} - n}{p}$, where m, n and p are positive integers and m is not divisible by the square of any prime, find the value of 100m + 10n + p.

29. One of the roots of the equation
$2000x^6 + 100x^5 + 10x^3 + x - 2 = 0$ is of the form $\frac{\sqrt{m} - n}{r}$, where m is non zero integer and n and r are relatively prime natural numbers. Find the value of m + n + r.

30. In a GP the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Find the number of terms in the G.P.

31. Given a three digit number whose digits are three successive terms of a G.P. If we subtract 792 from it, we get a number written by the same digits in the reverse order. Now if we subtract four from the hundred's digit of the initial number and leave the other digits unchanged, we get a number whose digits are successive terms of an A.P. Find the number.